Five daily lessons

National Numeracy Strategy

Unit 12
Properties of numbers and number sequences

Year 5
Summer term

This Unit Plan is designed to guide your teaching.
You will need to adapt it to meet the needs of your class.

Unit Objectives

Year 5

- Make general statements about odd or even numbers, including the outcome of sums and differences. — Page 19
- Find all the pairs of factors of any number up to 100. — Page 21
- Make and investigate a general statement about familiar numbers or shapes by finding examples that satisfy it. Explain a generalised relationship (formula) in words. — Page 81

Link Objectives

Year 4

- Recognise odd and even numbers up to 1000 and some of their properties, including the outcome of sums or differences of pairs of odd/even numbers.
- Recognise multiples of 2, 3, 4, 5 and 10 up to the tenth multiple.
- Make and investigate a general statement about familiar numbers or shapes by finding examples that satisfy it.

(Key objectives in bold)

Year 6

- Make general statements about odd or even numbers, including the outcome of products.
- Recognise prime numbers to at least 20. Factorise numbers to 100 into prime factors.
- Make and investigate a general statement about familiar numbers or shapes by finding examples that satisfy it. Develop from explaining a generalised relationship in words to expressing it in a formula using letters as symbols (e.g. the cost of *n* articles at 15p each).

Resources needed to teach this unit:

- Activity sheet 12.1
- OHT 12.1
- OHT 12.2
- Resource sheet 12.1
- Self-assessment sheet 12.1
- Whiteboards
- NNS Interactive Teaching Programme 'Number Grid' or displayed 100 number grid
- Hundred squares

department for education and skills

Planning sheet	Day One	Unit 12 *Properties of numbers and number sequences*	Term: *Summer*	Year Group: *5*

Oral and Mental

Objectives and Vocabulary	Teaching Activities
Find factors of numbers to 100. VOCABULARY multiple factor RESOURCES Whiteboards OHT 12.1	• Write on the board multiple, factor **Q** Can anyone remember and explain what we mean by the terms multiple and factor? • Discuss responses and establish that a factor is any whole number which divides exactly into another whole number, e.g. 3 is a factor of 12 because it divides into 12 exactly 4 times, 35 is a multiple of 5, because 5 multiplied by 7 is 35. Show OHT 12.1. **Q** Can you see any factors of 24? The children show these on their whiteboards. Establish answers and correct any misunderstandings. Continue with further questions. **Q** What factors of 27 can you see? What factors of 100 are there? Can you see any factors of 42? What factors of 64 can you see? • Collect and discus answers and correct any misconceptions.

Main Teaching

Objectives and Vocabulary	Teaching Activities
Make and investigate a general statement about familiar numbers by finding examples that satisfy it. Express a relationship orally and in writing. VOCABULARY odd even prime consecutive multiple factor diagonal RESOURCES NNS interactive teaching programme 'Number grid' or hundred number grid Resource sheet 12.1	• Launch the number grid interactive teaching programme or display a hundred number square and highlight two consecutive numbers, e.g. 15 and 16. **Q** What is the sum of these two numbers? Establish the answer and then invite the children to choose and highlight other pairs of consecutive numbers. Each time, ask the class to find the total. **Q** Can you see a pattern and give a rule to find the total using only one of the numbers? Collect suggestions. • Make the general statement: The sum of two consecutive numbers is double the first number and add 1. Ask the children to try this out for other pairs on the grid. Collect and display some responses e.g. **Number pairs** / **Total** 15, 16 / 31 9, 10 / 19 33, 34 / 67 47, 48 / 95 **Q** Do all our examples satisfy our general rule? Agree they do. **Q** Can anyone describe another rule for finding the sum of any two consecutive numbers? Discuss responses. Establish that double the second number then subtract 1 also works. Help the children to recognise why these two rules will always work. • Highlight any 2 x 2 window on the grid, e.g. 6 / 7 16 / 17 Ask the children to describe the relationship between the four numbers using, e.g. 1 less, 10 more, etc. **Q** What do you notice about the total of the two pairs of diagonal numbers? Establish that 6 + 17 = 23 and 16 + 7 = 23. **Q** Can you make general statements about numbers on a 2 x 2 window that we can test? Collect and record suggestions. • Give out Resource sheet 12.1. Ask the class to work in pairs to test the general statements on other 2 x 2 windows on the grid and record their results in their books. • Collect results and discuss the general statements. Help the children to see how the structure in the table leads to the property. • Discuss the extension to other sizes of windows, e.g. 3 x 3, and ask the children to make general statements they can test. **10** / 11 / **12** 20 / 21 / 22 **30** / 31 / **32** Collect and discussion the children's statements and observations.

Plenary

Teaching Activities/Focus Questions

• Highlight any 2 x 2 window on the hundred grid.

Q If we know the number in the top left-hand corner, what can we always say about the number to the right of it?

Discuss responses and establish that this is always 1 more.

Q What can we say about the number that is immediately below the number in the top left-hand corner?

Establish that this is always 10 more.

Q What can we say about the number that is in the bottom right-hand corner?

Establish that this is always 11 more. Confirm this with other windows.

• Summarise this in words as:

Number	**Number plus 1**
Number plus 10	**Number plus 11**

• Use the table to explain the total of a diagonal is always twice the first number plus 11.

By the end of the lesson the children should be able to:

- **Find examples which match a general statement about familiar numbers by finding examples that satisfy it.**

(Refer to supplement of examples, section 6, page 81.)

Planning sheet	Day Two (page 1 of 2)	Unit 12 *Properties of numbers and number sequences*	Term: *Summer*	Year Group: *5*

Oral and Mental

Objectives and Vocabulary

Consolidate mental calculation strategies and place value using imagery of a hundred square.

VOCABULARY
sum
total

RESOURCES
Whiteboards
Hundred square

Teaching Activities

- Ensure that there are no hundred squares visible for this part of the lesson. Ask the class to visualise a hundred square.

Q If you start on 64 and move one row down what number do you get to?

Ask the children to show their answer on their whiteboards.

- Continue to ask questions such as:

Q What number is two rows directly below 37?
Which number is three rows above 83?
Which number is seven places to the right of 23?
Which number is four places to the left of 49?
If I move two lines down from 48 what would I be adding to 48?
Start at 24 and move one place to the right and two rows down. Where do you get to?
Starting at 19, which move takes you to 57? Is there another way?

- Discuss responses. Now show a hundred square and confirm the correct answers.

Main Teaching

Objectives and Vocabulary

Make and investigate a general statement about familiar numbers by finding examples that satisfy it.

Express a relationship orally and in writing.

VOCABULARY
sum
total
consecutive

RESOURCES
NNS Interactive Teaching Programme 'Number grid' or a hundred number grid
Whiteboards
Resource sheet 12.1

Teaching Activities

- Launch the number grid interactive teaching programme or display a hundred number square. Highlight any three consecutive numbers, e.g. 24, 25, 26.

Q What can we say about these numbers?

Discuss responses and establish that the numbers are consecutive.

Q What is the sum of these three numbers?

Collect answers and discuss methods.

- Invite children to choose other sets of three consecutive numbers. Ask the class to work out the total and display this on their whiteboards.

- Record findings in a table and use the information to make general statements.

Three consecutive numbers	Total
24, 25, 26	75
3, 4, 5	12
9, 10, 11	30
31, 32, 33	96
49, 50, 51	150

Q If we know the middle number, how can we find the total?

Agree It is the middle number multiplied by 3. Record on the board:

72	73	74	→	73	73	73

Explain that 72 + 73 + 74 = 73 + 73 + 73 as the loss of 1 from 73 to get 72 is compensated for by the addition of 1 to get 74.

Q Will this always be the case?

Use further examples to highlight the pattern and the argument. Establish that it will always work. Record on the board:

N -1	N	N + 1	→	N	N	N

Say that N stands for the middle number so N – 1 is 1 less than N and N + 1 is 1 more that N. In this case, (N – 1) + N + (N + 1) = N + N + N.

Say for shorthand N + N + N is 'three times N' or '3N'. Emphasise that using the letter N for a number helps us to see the patterns in the numbers.

- Highlight a window on the shape of a cross, e.g.

	3	
12	13	14
	23	

Plenary

Teaching Activities/Focus Questions

- Ask the children:

Q What did you discover about the cross and the z windows?

Establish that in all cases the total is five times the middle number.

Q For the cross what can we say about the numbers directly above and below the middle number?

Establish that these are 10 more and 10 less than the middle number.

Q What can we say about the numbers that are directly to the right and left of the middle number?

Establish that these are 1 more and 1 less than the middle number.

- Draw on the board:

	35		→		45			
44	45	46		45	45	45		
	55				45			

Establish that you can replace each number with a 45 because of the 1 more and less horizontally, and the 10 more and less vertically. So: 35 + 44 + 45 + 46 + 55 = 45 + 45 + 45 + 45 + 45 = 5 x 45 = 225.

Use further examples to highlight the pattern and argument.

- With the class establish how the pattern can be represented using N for number.

	N - 10		→		N			
N - 1	N	N + 1		N	N	N		
	N + 10				N			

Agree that:
(N – 10) + (N – 1) + N + (N + 1) + (N + 10) = N + N + N + N + N = 5N.

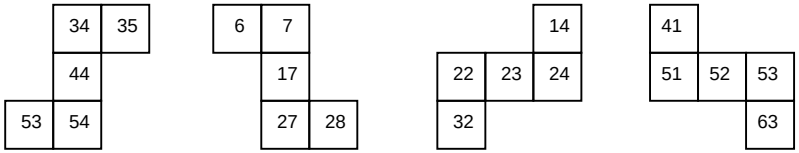
By the end of the lesson the children should be able to:
- **Make and investigate a general statement about familiar numbers;**
- **Express a relationship orally and in writing.**

(Refer to supplement of examples, section 6, page 81.)

Planning sheet	Day Two (page 2 of 2)	Unit 12 *Properties of numbers and number sequences*		Term: *Summer*	Year Group: *5*
Oral and Mental		**Main Teaching**		**Plenary**	
Objectives and Vocabulary	**Teaching Activities**	**Objectives and Vocabulary**	**Teaching Activities**	**Teaching Activities/Focus Questions**	

Q What do the five numbers in the cross sum to?

Agree it is 65. Give out Resource sheet 12.1. Tell the children they are to work in small groups and find other 'cross totals', record their results, and make a general statement then test it and try to find an explanation as to why their general statement is true. Say that after the cross they can use a Z window from one of the following:



Planning sheet	**Day Three**	**Unit 12 *Properties of numbers and number sequences***	**Term: *Summer***	**Year Group: *5***

Oral and Mental		**Main Teaching**		**Plenary**
Objectives and Vocabulary	**Teaching Activities**	**Objectives and Vocabulary**	**Teaching Activities**	**Teaching Activities/Focus Questions**
Recall properties of numbers. VOCABULARY odd even consecutive square multiple factor prime RESOURCES Whiteboards	• Ask quick fire questions about properties of numbers. **Q** Can you show me: an even number greater than 400? an odd number less than 59? two consecutive numbers between 49 and 57? an odd multiple of 3? a factor of 24? an even multiple of 7? a square number less than 70? a prime number greater than 10 and less than 20? • The children respond using whiteboards. • Discuss and share responses and ensure that the children understand the vocabulary.	Make a general statement about odd and even numbers and/or give examples that match them. VOCABULARY sum difference odd even RESOURCES NNS Interactive Teaching Programme 'Number grid' or a 100 number grid Activity sheet 12.1 Resource sheet 12.1 OHT 12.2	• Launch the interactive teaching programme number grid or display a 100 number grid. Highlight multiples of 2. **Q** What do we call these numbers? Agree they are the even numbers. **Q** If I add any two even numbers together do I get an odd number or an even number? Discuss responses and confirm with examples, e.g. 2 + 4 even, 10 + 22 even, 54 + 58 even, 22 + 76 even. Establish that the sum of two even numbers is even. **Q** What do we call the remaining numbers on the grid that are not highlighted? • Establish that these are the odd numbers. **Q** Would we get the same result if we added any two odd numbers? • Discuss responses and encourage the children to justify their answers with examples, e.g. 3 + 5 = 8 even, 37 + 39 = 76 even. • Tell the class that today they will be finding other general rules for adding odd and even numbers. Give out Activity sheet 12.1 and Resource sheet 12.1 and ask the children to work with partners to complete the general statements and give at least four examples that meet each statement. If they complete the sheet they are to find their own general statements and provide examples to test them.	• Collect the children's responses and examples. For each ask: **Q** Can you explain why the statement might be true? • Discuss responses and establish that for example: Even numbers always end in 0, 2, 4, 6 or 8 and if you add 0, 2, 4, 6 or 8 to 0, 2, 4, 6, 8 you still get a number ending in 0, 2, 4, 6, 8. Odd numbers end in 1, 3, 5, 7, 9 and if you add 1, 3, 5, 7 or 9 to 1, 3, 5, 7 or 9 you get a number which ends in 2, 4, 6, 8 or 0, an even number. If you add an odd and an even number you will be adding 1, 3, 5, 7 or 9 to a number ending in 0, 2, 4, 6 or 8 which will give a number ending in 1, 3, 5, 7 or 9, in other words an odd number. **By the end of the lesson the children should be able to:** • **Explore and give some examples to satisfy these general statements; the sum of three even numbers is even; the sum of three odd numbers is odd; the difference between one odd and one even number is odd, the difference between two odd or two even numbers is even.** (Refer to supplement of examples, section 6, page 19.)

Planning sheet	Day Four	Unit 12 *Properties of numbers and number sequences*	Term: *Summer*	Year Group: *5*

Oral and Mental

Objectives and Vocabulary	Teaching Activities
Recall properties of number and multiplication and division facts. VOCABULARY venn diagram factor multiple less than intersection	• Draw a Venn diagram on the board with two circles inside a rectangle. Decide secretly on two criteria, e.g. odd and less than 50, and multiples of 5. • Ask the children to give you a number less than 100. As it fits one of the criteria write it in the appropriate circle, taking care with numbers in the overlap. Write numbers which do not fit either criteria in the rectangle outside the circles. Stop at various points and ask: **Q** What two rules am I using to decide where I place the numbers? Take suggestions and ask the children to explain their thinking. If they are right they can give you two new criteria or rules. • Repeat the activity two or three times. Each time ask: **Q** What do the numbers in the intersection represent? Establish that these are numbers that meet both rules, e.g. odd numbers which are less than 50 and also multiples of 5.

Main Teaching

Objectives and Vocabulary	Teaching Activities
Find all the factor pairs of numbers up to 100. VOCABULARY factor factor pair RESOURCES Self-assessment sheet 12.1	• Ask the children to write down as many factors of 24 as they can. **Q** How many factors should there be? How can you be sure you have all the factors? Discuss responses then tell the class that you are going to show them a systematic method of finding factors which ensures that they do not miss any. Demonstrate the method of finding factor pairs for 24. **Q** Which factors can we write down straight away? Establish that all numbers have 1 and themselves as factors so we can write down 1 and 24 straightaway. Record 1, 24 (1 x 24 = 24). **Q** Is 2 a factor of 24? How do you know? Establish that all even numbers have 2 as a factor and as 24 ends in 4, it is even so 2 is a factor. **Q** What is 2 multiplied by to make 24? Establish that this is 12, so 2 and 12 are also a factor pair of 24. Record 2, 12 (2 x 12 = 24). **Q** What number could we try next? Is 3 a factor of 24? Establish that 3 is a factor of 24 because 24 is in the 3 times table. Agree that 8 x 3 = 24. Record 3, 8 (3 x 8 = 24). Continue with similar questions until all the factor pairs of 24 have been established. Write the factors out in a list: 1 2 3 4 6 8 12 24 Identify the factor pairs 1 and 24, 2 and 12, 3 and 8, 4 and 6. **Q** Did we need to try any more factors after 4? • Discuss responses. Establish that 5 is not a factor of 24 and the next factor is 6. As 6 x 4 = 24, this would give us 6 and 4 as a factor pair but we already have 4 and 6, which is the same pair. Once we start repeating factors we know we do not need to go any further. • Repeat using 56. Now give the children sets of numbers to find all the factor pairs using this method, e.g. 15, 28, 40, 25, 19, 54, 21, 23, 33, 64, 80, 100, 92, 36, 63.

Plenary

Teaching Activities/Focus Questions

• Collect answers.

Q Which numbers had three or more pairs of factors?

Discuss responses and agree these numbers.

Q Which numbers had only one pair of factors? What is special about these numbers?

• Establish that there are numbers such as 19, 23. These are prime numbers which only have themselves and 1 as factors.

ASSESSMENT – Ask the children to complete the first cloud question on Self-assessment sheet 12.1, discuss it with a friend as before and tick the appropriate box. Help those children who need it.

HOMEWORK –

• Tell the children that the Greek mathematician Euclid who lived from 330 to 275 BC discovered 'perfect numbers'.

A perfect number is a number which is equal to the sum of its proper factors, i.e. all factors apart from the number, e.g. 6 is a perfect number because its proper factors 1, 2 and 3 sum to itself, i.e. 6 = 1 + 2 + 3.

Their task is to try and find the next perfect number after 6. They need to test numbers greater than 6, adding all the factors, apart from the number itself, to see if their total is equal to the number. Give a clue, you could tell them that the number is below 50. (The next perfect number is in fact 28, then 496).

By the end of the lesson the children should be able to:

- **Find all the pairs of factors of any number up to 100.**

(Refer to supplement of examples, section 6, page 21.)

Planning sheet	Day Five (page 1 of 2)	Unit 12 *Properties of numbers and number sequences*	Term: *Summer*	Year Group: *5*

Oral and Mental

Objectives and Vocabulary

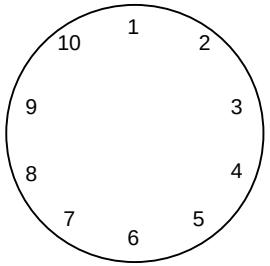
Recognise and explain patterns and relationships, generalise and predict.

VOCABULARY
rule
relationship
predict

RESOURCES
Whiteboards

Teaching Activities

- On the board draw the dial:



Say that you are thinking of a rule. Ask the children for one of the numbers on the dial and write next to it, outside the dial, the output number. Ask children to identify the rule. Set them the challenge of finding the rule before all ten numbers are used. Start with the rule multiplying by 3. Record the rules when the children guess them correctly in a table:

IN	OUT
N	3N

Explain that N is the number in the dial. The rule describes how to find the numbers on the outside of the dial. Repeat with other multiples.

- Explain that the next rule is a two-step rule. In the table write N and 3N – 1. Explain the rule and confirm that the children can use it to find numbers on the outside of the dial for given numbers in the dial. Repeat using other two-step rules.

Main Teaching

Objectives and Vocabulary

Recognise and explain patterns and relationships, generalise and predict.

VOCABULARY
rule
relationship
predict
inverse

RESOURCES
Whiteboards
OHT 12.2
Self-assessment sheet 12.1

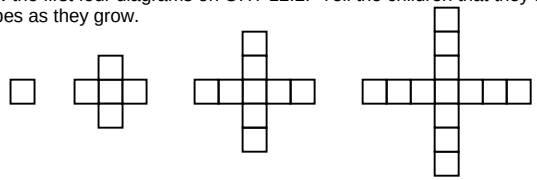
Teaching Activities

- Refer to the previous day's homework. Remind the class of the definition of a perfect number.

Q What is the next perfect number after 6?

Discuss the children's answers. Ask a child to demonstrate why 28 is a perfect number i.e. its proper factors (not including itself) are: 1, 2, 4, 7, 14 and 1 + 2 + 4 + 7 + 14 = 28. Say the next perfect number is 496 and then they get much bigger, very quickly.

- Show the first four diagrams on OHT 12.2. Tell the children that they are going to explore what happens to shapes as they grow.



Shape Number	Number of Squares
1	1
2	5
3	9
4	13

Q How many squares will shape number 5 have? How do you know?

Establish there will be 17 squares and each time 4 is added to the previous total. Add more data to the table. For shape number 6 confirm the answer is 21 referring to the shapes on the OHT.

Q How many squares would be in shape number 10? How can we work this out without carrying on the table up to shape number 10?

Discuss responses. Encourage the children to look for a relationship in the table between shape number and squares just as they did in the oral and mental starter with inside and outside numbers on the dials.

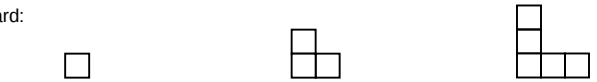
Q Can you explain this relationship in words?

- Collect and discuss suggestions. Establish that the number of squares is four times the shape number subtract 3. Write this on the board as Squares = 4 x shape number – 3.
Explain that if we use N for the shape number we can write this in short hand as Squares = 4N – 3.
Now get the class to use this rule to work out the number of squares for some different shape numbers, e.g. in shape number 10 there are 40 – 3 = 37 squares.

Q How many squares will shape number 20 have? What number shape would use 29 squares?

Discuss responses and address any misconceptions. Ensure that the children understand that for the last question they need to work backwards.

- Draw on the board:



Shape Number 1 Shape Number 2 Shape Number 3

Plenary

Teaching Activities/Focus Questions

- Set up a results table on the board for the T shape.

Shape Number	Number of Squares
1	1
2	4
3	7
4	

Establish that the relationship this time between the shape number and number of squares is:
Number of Squares = 3 x shape number – 2 or in shorthand: Squares = 3N – 2.

Q How could we work out the number of squares for the 20th shape?

Q How can we use this rule to work out the shape number which has 31 squares?

Collect and discuss answers.

ASSESSMENT –

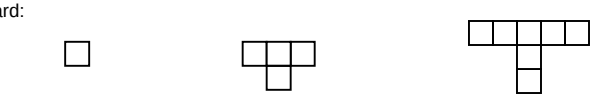
- Ask the children to complete the second and third cloud questions on Self-assessment sheet 12.1 and tick the appropriate box. Help those children who need it.

Remind the children of the work hey have been engaged in during the week. Ask them to identify what mathematics they want to get better at and to record this as a target on the sheet.

By the end of the lesson the children should be able to:

- **Express a relationship in words, orally and in writing;**
- **Begin to ask, 'what if?'.**

(Refer to supplement of examples, section 6, page 81.)

Planning sheet	Day Five (page 2 of 2)	Unit 12 *Properties of numbers and number sequences*	Term: *Summer*	Year Group: *5*
Oral and Mental		**Main Teaching**		**Plenary**
Objectives and Vocabulary	**Teaching Activities**	**Objectives and Vocabulary**	**Teaching Activities**	**Teaching Activities/Focus Questions**
			Ask the children to describe how the shapes grow, and how many squares there are in shape number 4, 5, 10, etc. Say that you want them to collect the information in a table then to find a rule using N for the shape number. • Collect and discuss. Establish that this time as there are two new squares added we multiply by 2. From the table agree that for this pattern the rule is: Squares = 2N – 1. Use this to check the values in the table and extend beyond these values. **Q** What number shape would use 99 squares? Discuss responses and establish that it would be shape number 50. • Draw on the board:  Shape Number 1 Shape Number 2 Shape Number 3 Ask the children to explore the growing shapes and to find a rule using N.	

Complete the following general statements. Using different sets of odd and even numbers, give at least four examples of numbers that meet the statement.

1. The sum of three even numbers is _____

2. The sum of three odd numbers is _____

3. An odd number added to an even number makes _____

4. The difference between an odd and an even number is _____

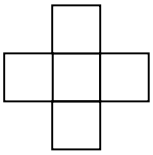
5. The difference between two odd numbers is _____

6. The difference between two even numbers is _____

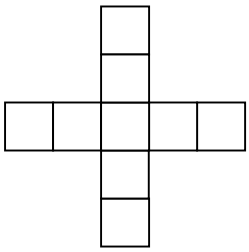
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150

27	35	100	8	5	21
2	42	4	12	20	25
9	6	16	40	15	32
10	7	3	14	18	50

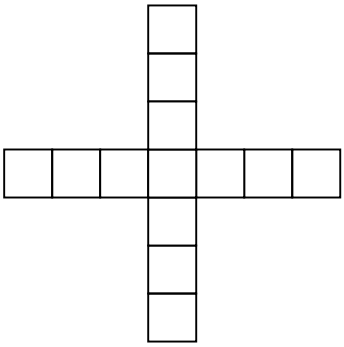
Shape Number 1



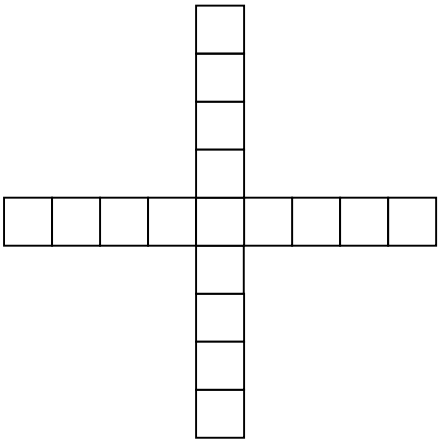
Shape Number 2



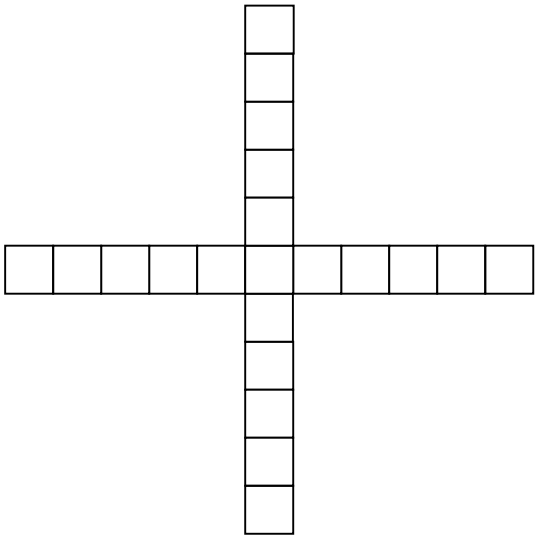
Shape Number 3



Shape Number 4



Shape Number 5



Shape Number 6

My Mathematics

Find all the factor pairs for:
60, 49, 108

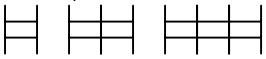
My calculation

60 49 108

Show or discuss with a friend

I did this:
on my own ☐
with some help ☐

The first shape needs four sticks. Complete the table showing the number of sticks needed for the shapes. Explain the rule, using N for the shape number



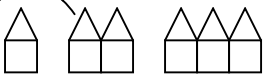
My table and rule

Shape Number	Number of sticks
1	4
2	
3	
4	
5	
6	
7	
8	
9	
10	

Show or discuss with a friend

I did this:
on my own ☐
with some help ☐

The first shape needs six sticks. Complete the table. Explain the rule using N for shape number. Use the rule to find the number of sticks needed for the 30th shape



My table and rule

Shape Number	Number of sticks
1	6
2	
3	
4	
5	
6	
7	
8	

Show or discuss with a friend

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with some help ☐

Name: _____

I want to get better at _____

_____